ECS455: Chapter 4

Multiple Access

4.4 DS/SS

Dr.Prapun Suksompong
prapun.com/ecs455

Office Hours:
BKD 3601-7
Tuesday 9:30-10:30
Friday 14:00-16:00

Spread spectrum (SS)

- Historically spread spectrum was developed for *secure* communication and *military* uses.
- **Difficult to intercept** for an unauthorized person.
- Easily **hidden**. For an unauthorized person, it is difficult to even detect their presence in many cases.
- **Resistant to jamming**.
- Provide a measure of immunity to distortion due to multipath propagation.
  - In conjunction with a RAKE receiver, can provide coherent combining of different multipath components.
- Asynchronous multiple-access capability.
- Wide bandwidth of spread spectrum signals is useful for location and timing acquisition.

[Goldsmith, 2005, Ch 13]

Spread spectrum: Applications

- First achieve widespread use in military applications due to
  - its inherent property of *hiding the spread signal below the noise floor* during transmission,
  - its resistance to narrowband jamming and interference, and
  - its low probability of detection and interception.
- The narrowband interference resistance has made spread spectrum common in cordless phones.
- The basis for both 2nd and 3rd generation cellular systems as well as 2nd generation wireless LANs.
  - The ISI rejection and bandwidth sharing capabilities of spread spectrum are very desirable in these systems

[Goldsmith, 2005, Ch 13]

Spread spectrum conditions

Spread spectrum refers to any system that satisfies the following conditions [Lathi, 1998, p 406 & Goldsmith, 2005, p. 378]:

1. The spread spectrum may be viewed as a kind of modulation scheme in which **the modulated (spread spectrum) signal bandwidth is much greater than the message (baseband) signal bandwidth.**
2. The **spectral spreading** is performed by a **code** that is **independent** of the message signal.
   - This same code is also used at the receiver to despread the received signal in order to recover the message signal (from the spread spectrum signal).
   - In secure communication, this code is known only to the person(s) for whom the message is intended.

[R. Pickholtz, D. Schilling, L. Milstein, "Theory of Spread-Spectrum Communications - A Tutorial," IEEE Trans. Commun., Vol. 30 , pp. 855-884, May 1982.]

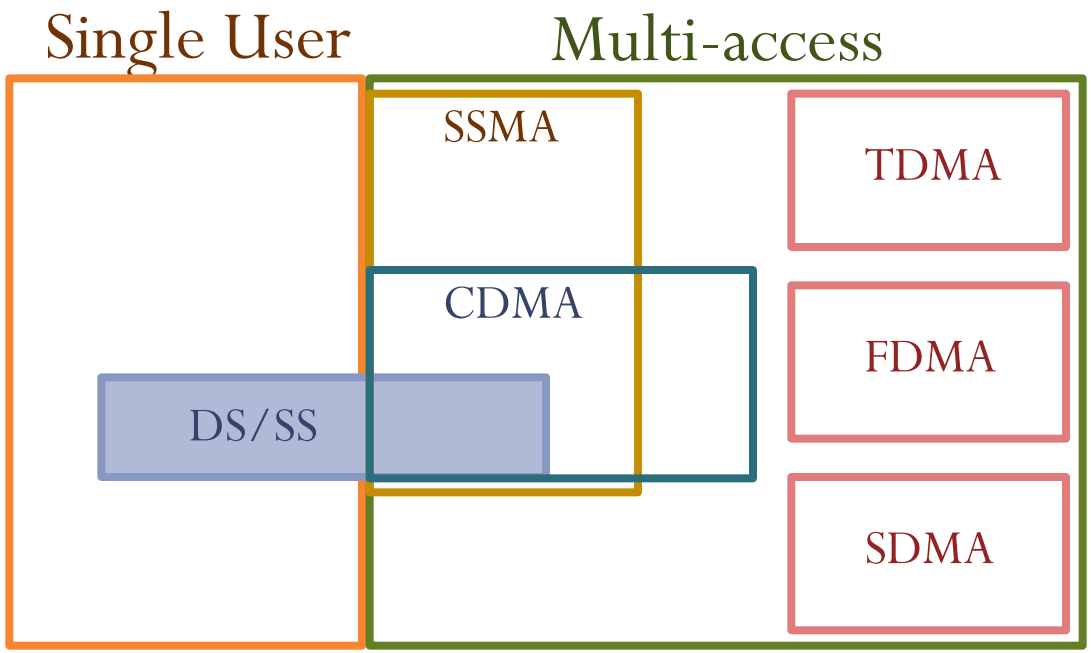
Spread spectrum (2)

- The spread spectrum scheme increases the bandwidth of the message signal by a factor *N*, called the **processing gain**.
  - In practice spread spectrum systems have processing gains on the order of 100-1000. [Goldsmith, 2005, p 379]
- Although we use much higher BW for a spread spectrum signal, we can also multiplex large numbers of such signals over the same band.
- Many users can share the same spread spectrum bandwidth without interfering with one another.
  - Achieved by assigning different code to each user.
  - Frequency bands can be reused without regard to the separation distance of the users.

IS-95 (CDMA)
N = 128 [T&V]

MU

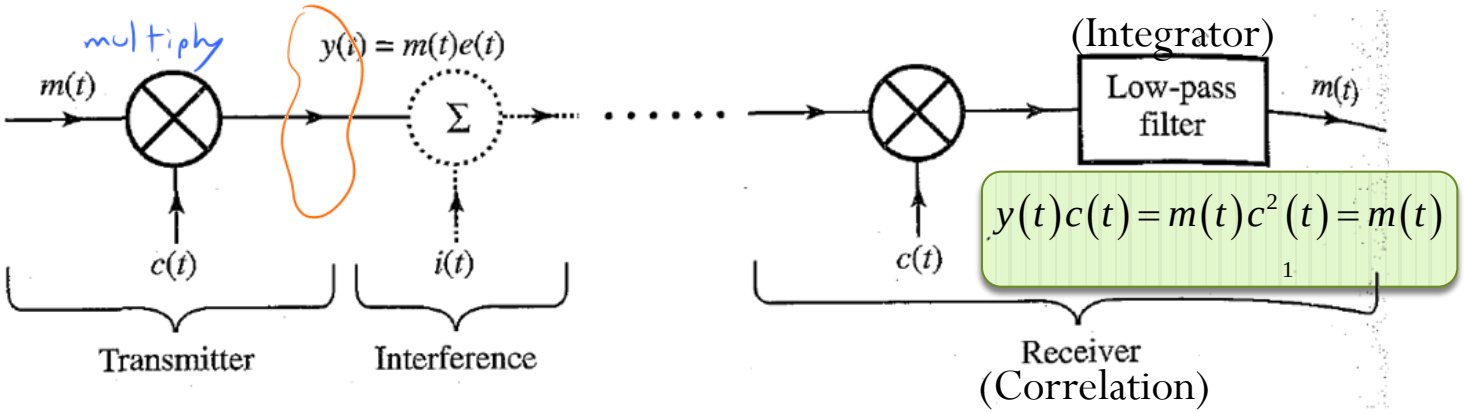
SSMA, CDMA, DS/SS



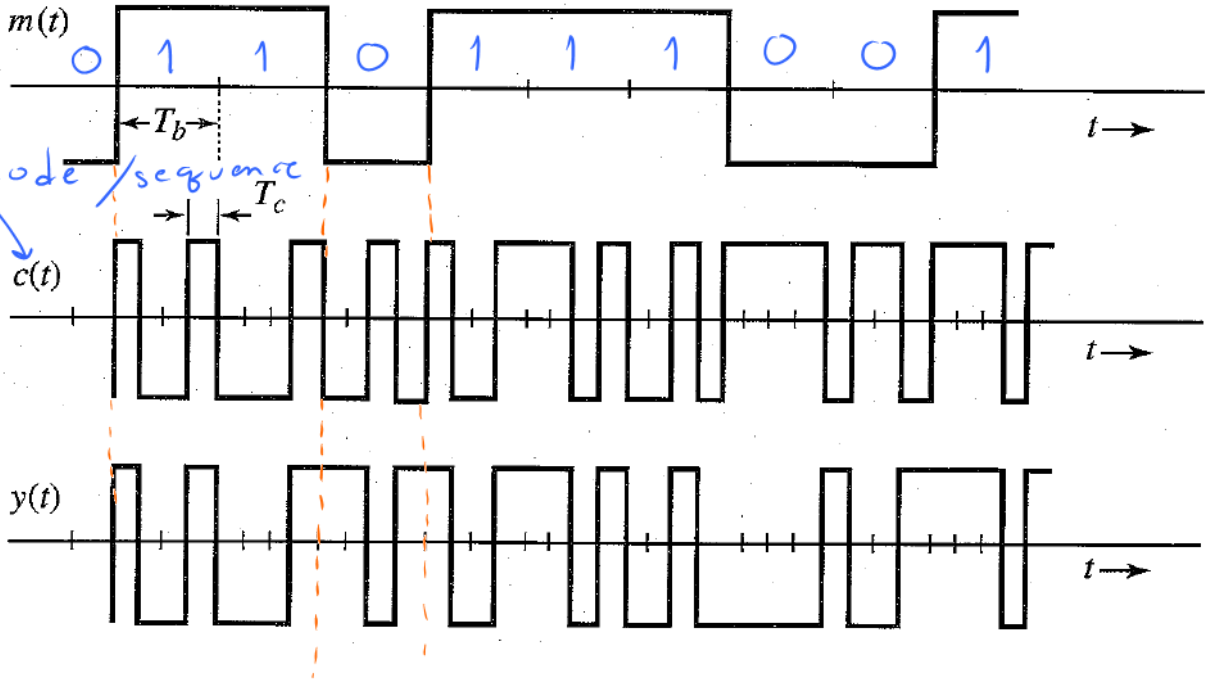
Direct Sequence

Useful even for single user!

DS/SS System



Message signal (polar binary signal)



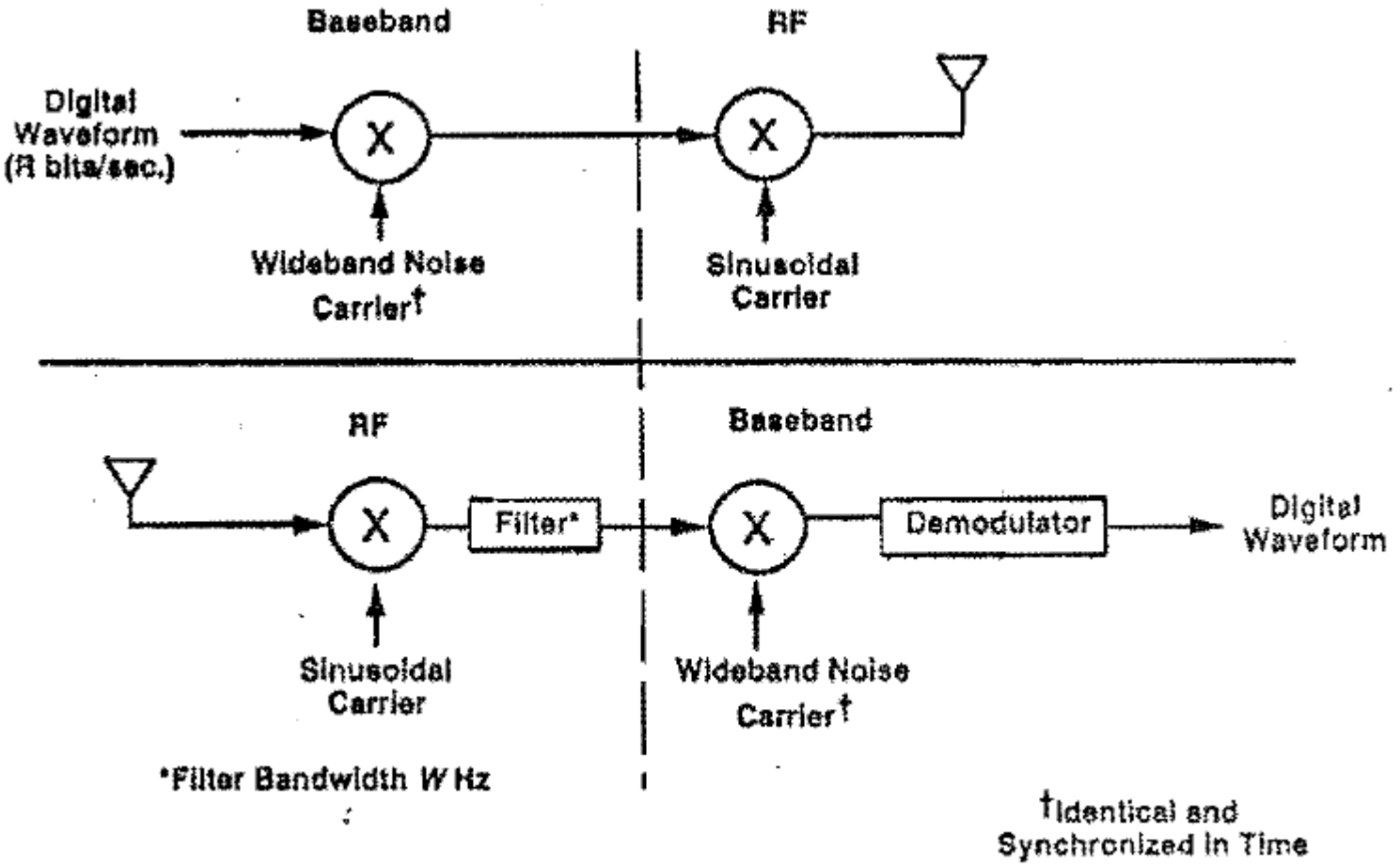
Polar signal representing **psuedonoise (PN) sequence.** (Think of this as a pseudorandom carrier)

$$\frac{T_b}{T_c} = N$$

DS/SS System (Con't)

- Notice that the process of detection (despreading) is identical to the process of spectral spreading.
  - Recall that for DSB-SC, we have a similar situation in that the modulation and demodulation processes are identical (except for the output filter).

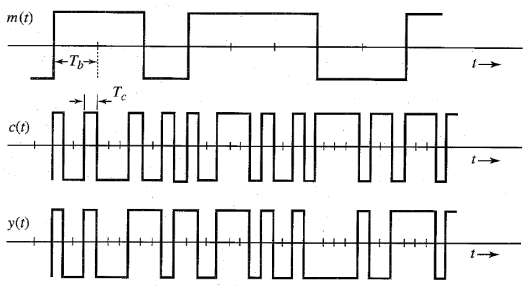
Spread spectrum modem



[Viterbi, 1995, Fig. 1.2]

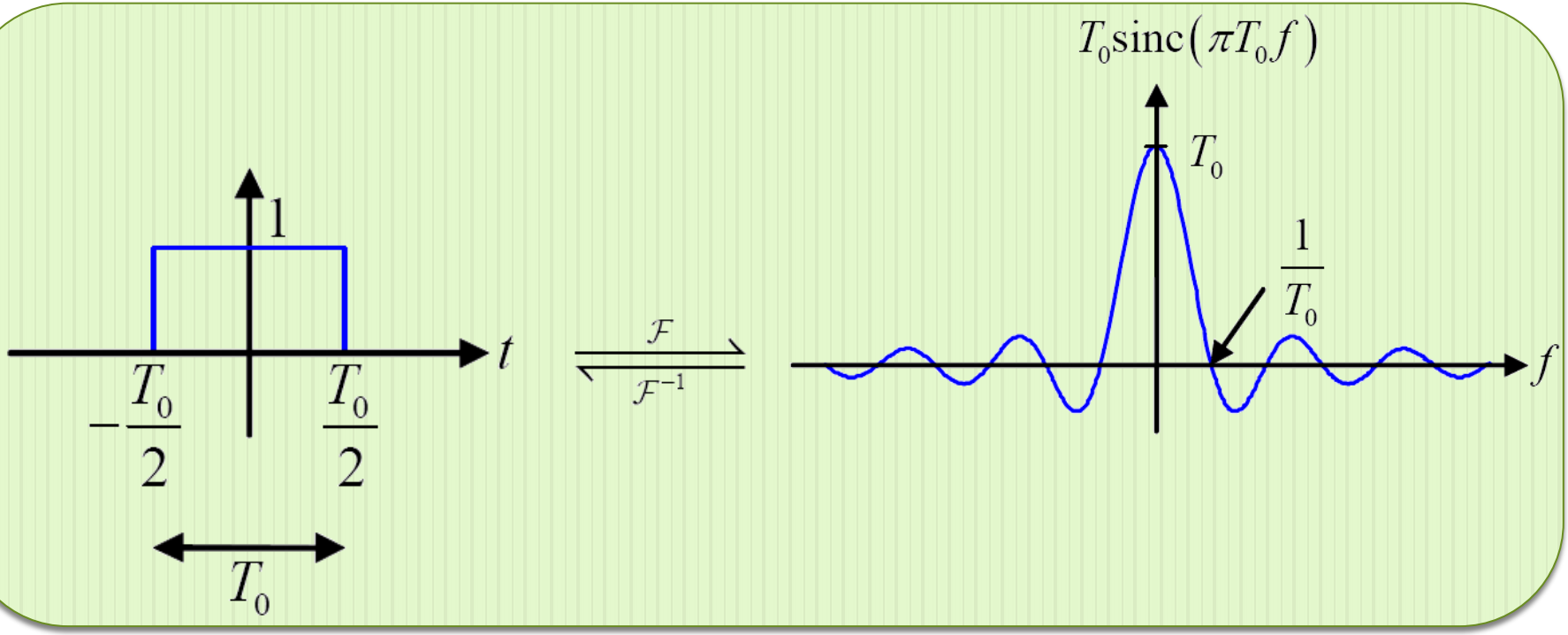
DS/SS: Spectral Spreading Signal c(t)

- A pseudorandom signal
  - Appear to be unpredictable
  - Can be generated by deterministic means (hence, pseudorandom)



- The bit rate is chosen to be much higher then the bit rate of m(t).
- The basic pulse in *c*(t) is called the **chip**.
- The bit rate of *c*(t) is known as the **chip rate**.
- The auto-correlation function of *c*(t) is very narrow.
  - Small similarity with its delayed version
- Remark: In multiuser (CDMA) setting, the cross-correlation between any two codes $c_1(t)$ and $c_2(t)$ is very small
  - Negligible interference between various multiplexed signals.

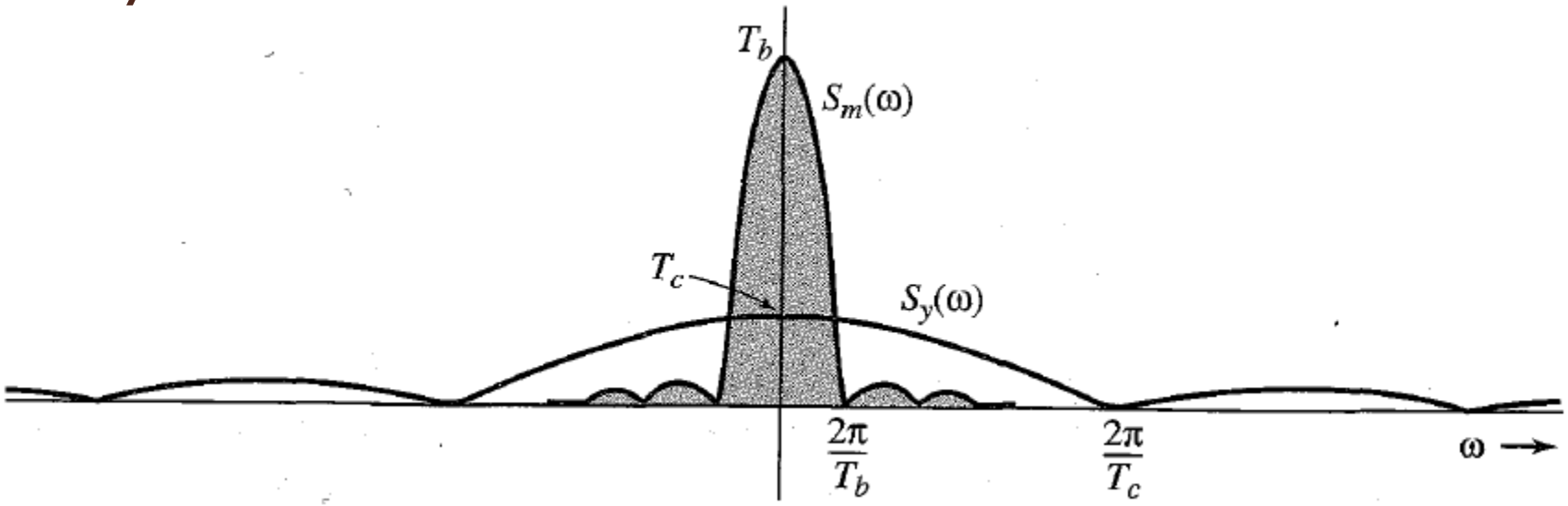
Frequency-Domain Analysis



Shifting Properties: $g(t-t_0) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j2\pi f t_0} G(f)$ $\quad e^{j2\pi f_0 t} g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f-f_0)$

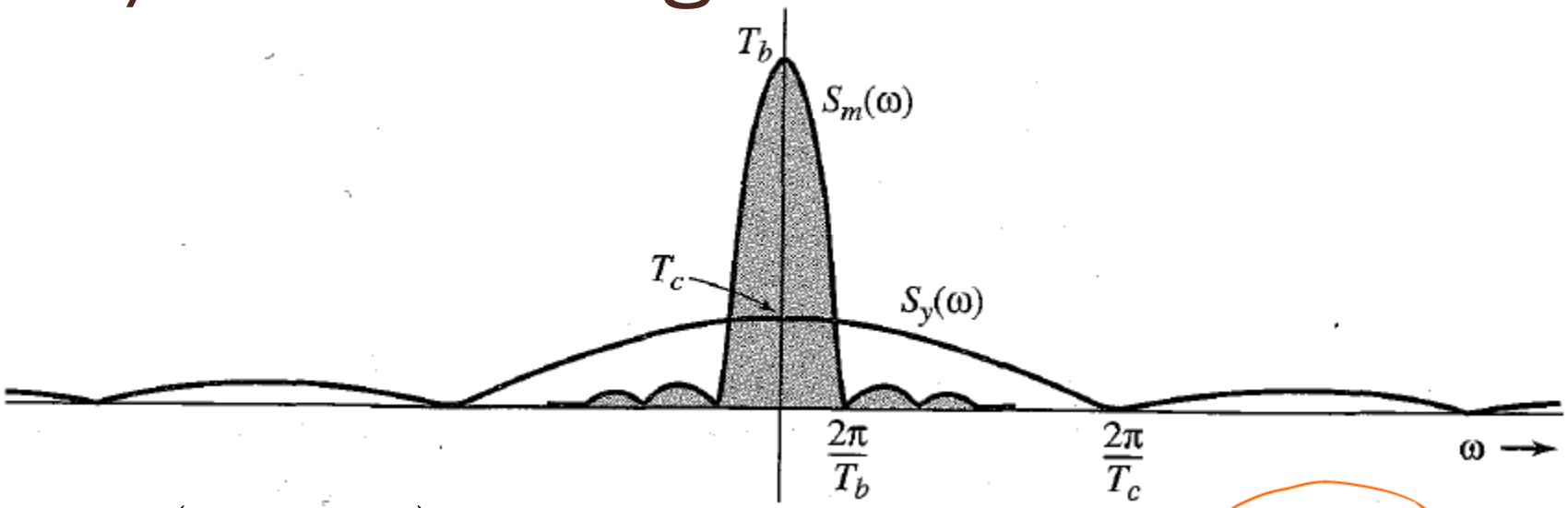
Modulation: $m(t)\cos(2\pi f_c t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{2}M(f-f_c)+\frac{1}{2}M(f+f_c)$

DS/SS: Secure Communication



- Secure communication
  - Signal can be detected only by authorized person(s) who know the pseudorandom code used at the transmitter.
  - Signal spectrum is spread over a very wide band, the signal PSD is very small, which makes it easier to hide the signal within the noise floor

DS/SS: Jamming Resistance



$$\big(y(t)+i(t)\big)c(t)=m(t)c^2(t)+i(t)c(t)=m(t)+i(t)c(t)$$

- Jamming Resistance / Narrowband Interference rejection
  - The decoder despreads the signal y(t) to yield m(t).
  - The jamming signal i(t) is spread to yield i(t)c(t).
  - Using a LPF, can recover m(t) with only a small fraction of the power from i(t).
- Caution: Channel noise will not spread.

DS/SS: Multipath Fading Immunity

- The signal received from any undesired path is a delayed version of the DS/SS signal.
- DS/SS signal has a property of low autocorrelation (small similarity) with its delayed version, especially if the delay is of more than one chip duration.
- The delayed signal, looking more like an interfering signal, will not be despread by $c(t)$ effectively minimizes the effect of the multipath signals.
- What is more interesting is that DS/SS cannot only mitigate but may also exploit the multipath propagation effect.
  - This is accomplished by a **rake receiver**.
  - This receive designed as to coherently combine the energy from several multipath components, which increases the received signal power and thus provides a form of diversity reception.
  - The rake receiver consists of a bank of correlation receivers, with each individual receiver correlating with a different arriving multipath component.
  - By adjusting the delays, the individual multipath components can be made to add coherently rather than destructively.

ECS455: Chapter 4

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4.5 m-sequence

Dr.Prapun Suksompong
prapun.com/ecs455

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Binary Random Sequences

- While DSSS chip sequences must be generated *deterministically*, properties of binary random sequences are useful to gain insight into deterministic sequence design.
- A random binary chip sequences consists of i.i.d. bit values with probability one half for a one or a zero.
  - Also known as Bernoulli sequences/trials, "coin-flipping" sequences
- A random sequence of length N can be generated, for example, by flipping a fair coin N times and then setting the bit to a one for heads and a zero for tails.

Pseudorandom Sequence

- **Key randomness properties** [Golomb, 1967]: Binary random sequences with length N asymptotically large have a number of the properties desired in spreading codes
  - **Balanced property** of a code: Equal number of ones and zeros.
  - **Run length property** of a code: The run length is generally short.
    - half of all runs are of length 1
    - a fraction $1/2^n$ of all runs are of length n (Geometric)
  - **Shift property** of a code: If they are shifted by any nonzero number of elements, the resulting sequence will have half its elements the same as in the original sequence, and half its elements different from the original sequence.
- A deterministic sequence that has the balanced, run length, and shift properties as it grows *asymptotically large* is referred to as a **pseudorandom sequence** (noiselike signal).

Don't want long runs

Note: A run is a subsequence of identical symbols within the sequence.

Pseudo random

Pseudonoise (PN) ~~signature~~ sequence

- Ideally, one would prefer a random binary sequence as the spreading sequence.
- However, practical synchronization requirements in the receiver force one to use periodic Pseudorandom binary sequences.
- m-sequences
- Gold codes
- Kasami sequences
- Quaternary sequences
- Walsh functions

Need to be
- easily implemented
- reproducible

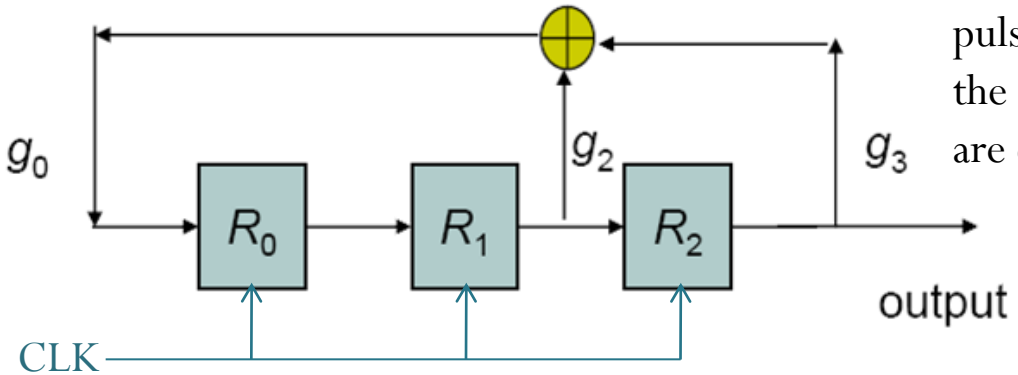
longer name: maximal length linear shift register sequence

m-Sequences

- **Maximal-length sequences**
- A type of **cyclic code**
  - Generated and characterized by a generator polynomial
  - Properties can be derived using algebraic coding theory
- Simple to generate with **linear feedback shift-register** (LFSR) circuits
  - Automated
- Approximate a random binary sequence in the sense that shifted versions of itself are approximately uncorrelated.
- Relatively easy to intercept and regenerate by an unintended receiver

m-sequence generator

- The feedback taps in the feedback shift register are selected to correspond to the coefficients of a **primitive polynomial**.



$g(x) = x^3 + x^2 + 1$ (Degree: r = 3)

Binary sequences drawn from the alphabet {0,1} are shifted through the shift register in response to clock pulses. The particular 1s and 0s occupying the shift register stages after a clock pulse are called **states**.

Time	R_0	R_1	R_2
0	1	0	0
1	0	1	0
2	1	0	1
3	1	1	0
4	1	1	1
5	0	1	1
6	0	0	1
7	1	0	0

Sequence repeats from here onwards

The g_i's are coefficients of a primitive polynomial.

1 signifies closed or a connection and
0 signifies open or no connection.

GF(2)

- **Galois field** (finite field) of two elements
- Consist of
  - the symbols 0 and 1 and
  - the (binary) operations of
    - **modulo-2** addition (XOR) and
    - **modulo-2** multiplication.
- The operations are defined by

$$0 \oplus 0 = 0, \quad 0 \oplus 1 = 1, \quad 1 \oplus 0 = 1, \quad 1 \oplus 1 = 0$$
$$0 \cdot 0 = 0, \quad 0 \cdot 1 = 0, \quad 1 \cdot 0 = 0, \quad 1 \cdot 1 = 1$$

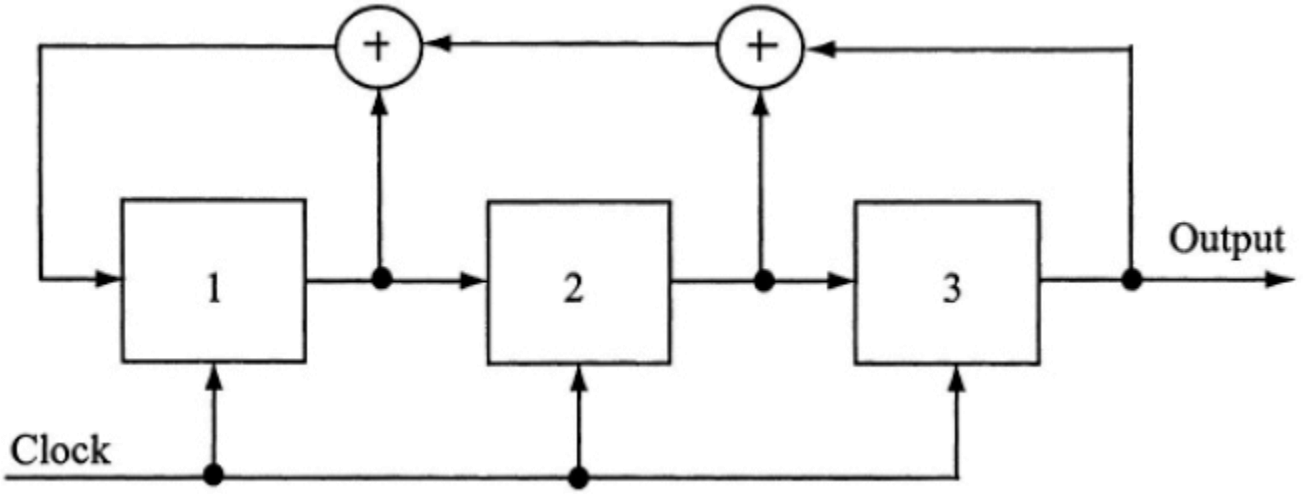
Sample Exam Question

Draw the complete **state diagrams** for linear feedback shift registers (LFSRs) using the following polynomials. Does either LFSR generate an m-sequence?

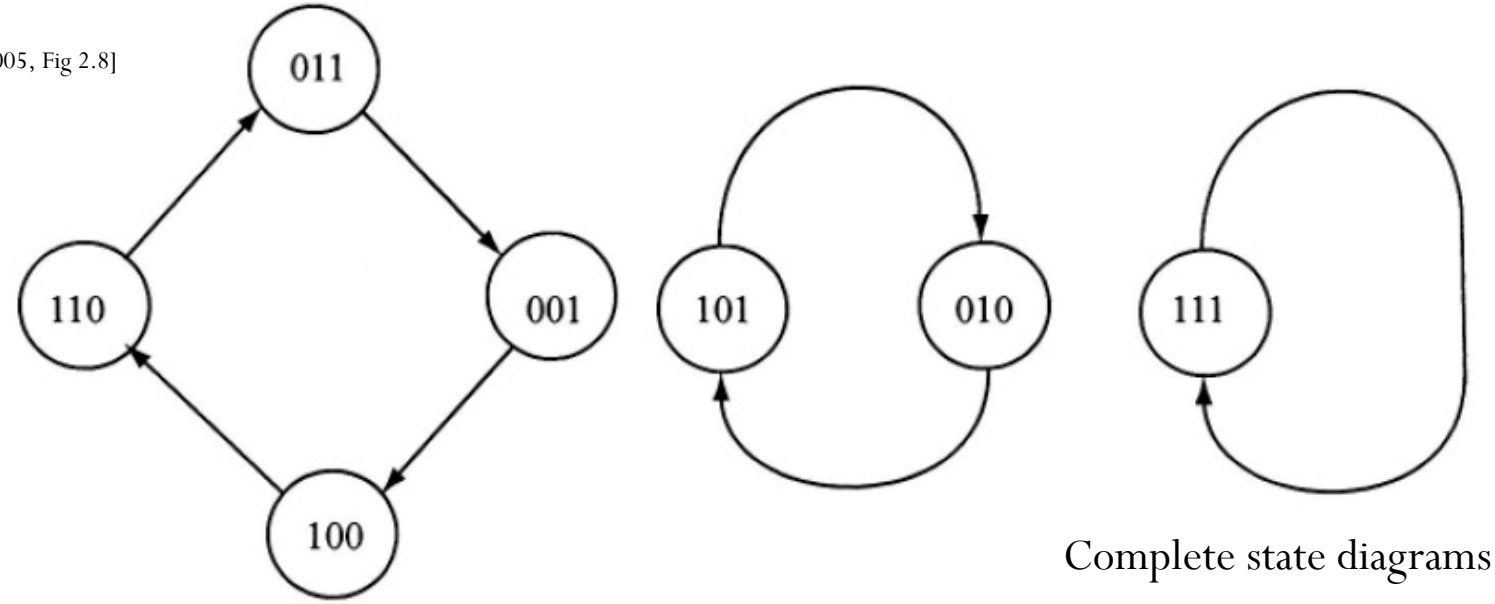
1. $x^3 + x^2 + 1$
2. $x^3 + x^2 + x + 1$

Nonmaximal linear feedback shift register

$$x^3 + x^2 + x + 1$$



[Torrieri , 2005, Fig 2.8]



Complete state diagrams

m-Sequences: More properties

1. The contents of the shift register will cycle over all possible 2^r-1 nonzero states before repeating.
2. Contain one more 1 than 0
3. Sum of two **(cyclic-)shifted** m-sequences is another (cyclic-)shift of the same m-sequence
4. If a window of width r is slid along an m-sequence for $N = 2^r-1$ shifts, each r-tuple except the all-zeros r-tuple will appear exactly once
5. For any m-sequence, there are
   - One run of ones of length r
   - One run of zeros of length r-1
   - One run of ones and one run of zeroes of length r-2
   - Two runs of ones and two runs of zeros of length r-3
   - Four runs of ones and four runs of zeros of length r-4
   - …
   - 2^{r-3} runs of ones and 2^{r-3} runs of zeros of length 1

order of primitive polynomial

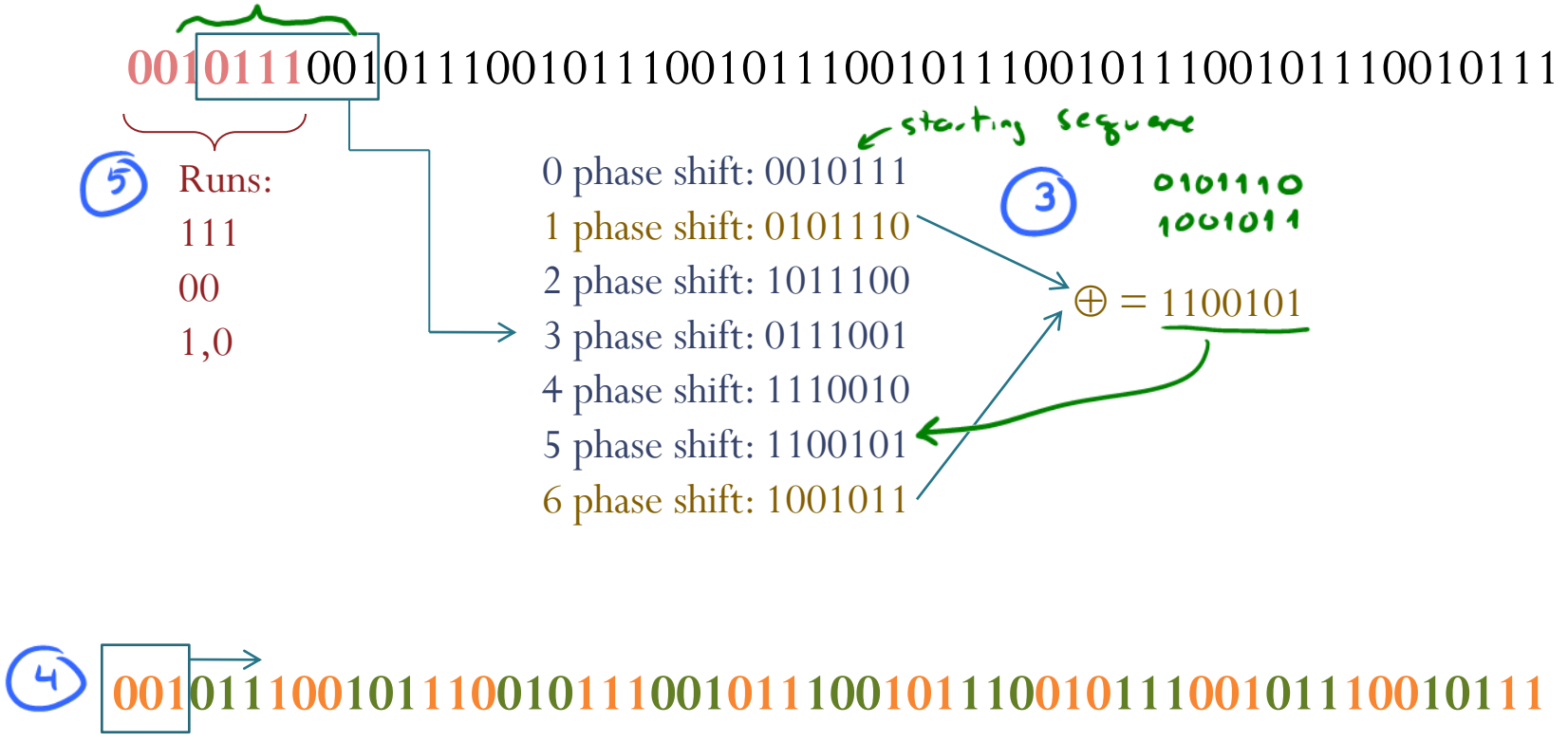
0010111 < 4 1s, 3 0s } ⇒ slightly unbalanced.

r-2 111111 r-2 000000

$2^{r-3} + 2^{r-3} = 2^{r-2}$

[S.W. Golomb, *Shift Register Sequences,* Holden-Day, San Francisco, 1967.]

Ex: Properties of m-sequence



Ex: Properties of m-sequence (con't)

r = 5

- 2^5-1 = 31-chip m-sequence

1010111011000111110011010010000

1010111011000111110011010010000

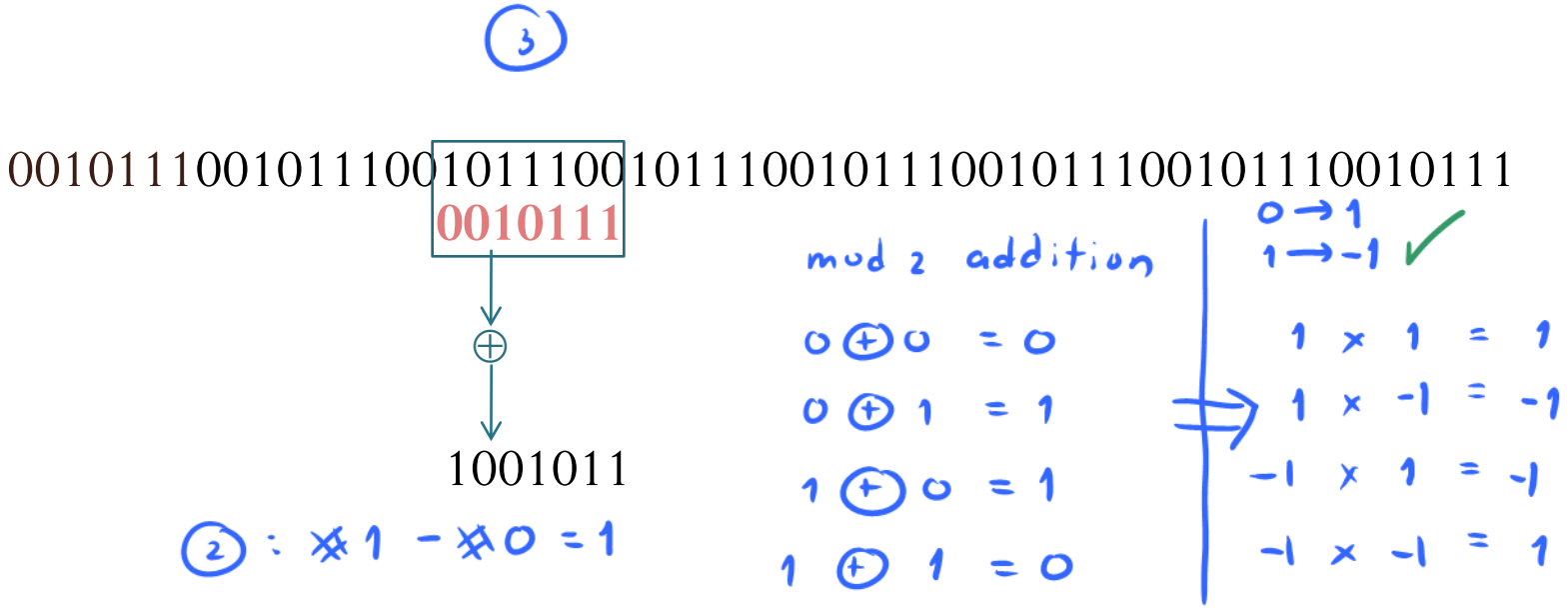
Runs:

11111	1
0000	1
111	1
000	1
11	2
00	2
1	4
0	4

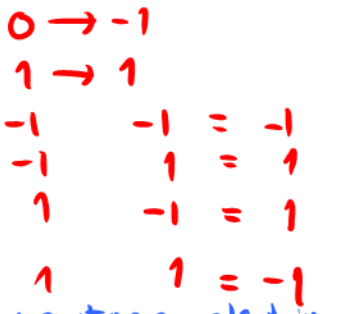
5

There are 16 runs.

m-Sequences (con't)



In actual transmission, we will map 0 and 1 to +1 and -1.



Correlation:

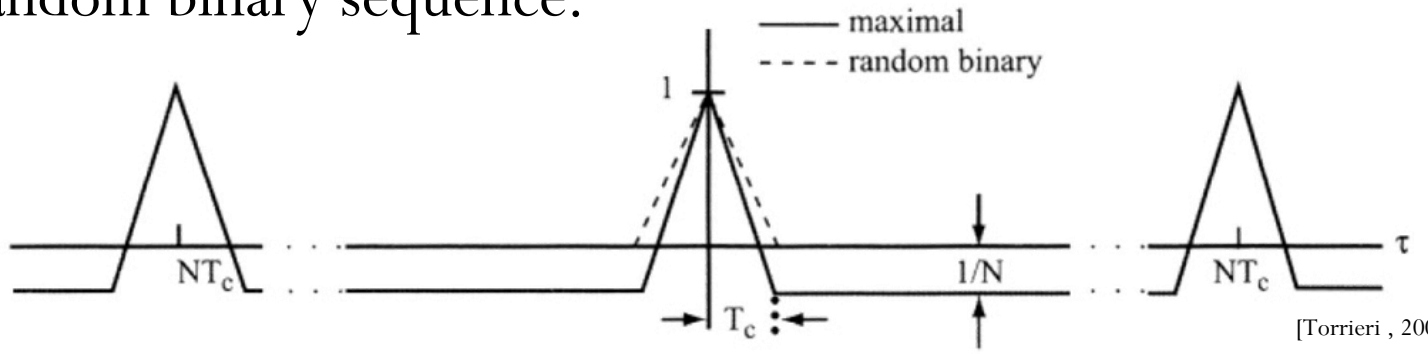
```
-1  1 -1 -1 -1  1  1
 1  1 -1  1 -1 -1 -1   ×
-------------------------
-1  1  1 -1  1 -1 -1
```

$\sum = -1$ v.s 7 : autocorrelation

$-\frac{1}{7}$ v.s. 1 : Normalized autocorrelation.

Autocorrelation and PSD

- (Normalized) autocorrelations of maximal sequence and random binary sequence.



[Torrieri , 2005, Fig 2.9]

- Power spectral density of maximal sequence.

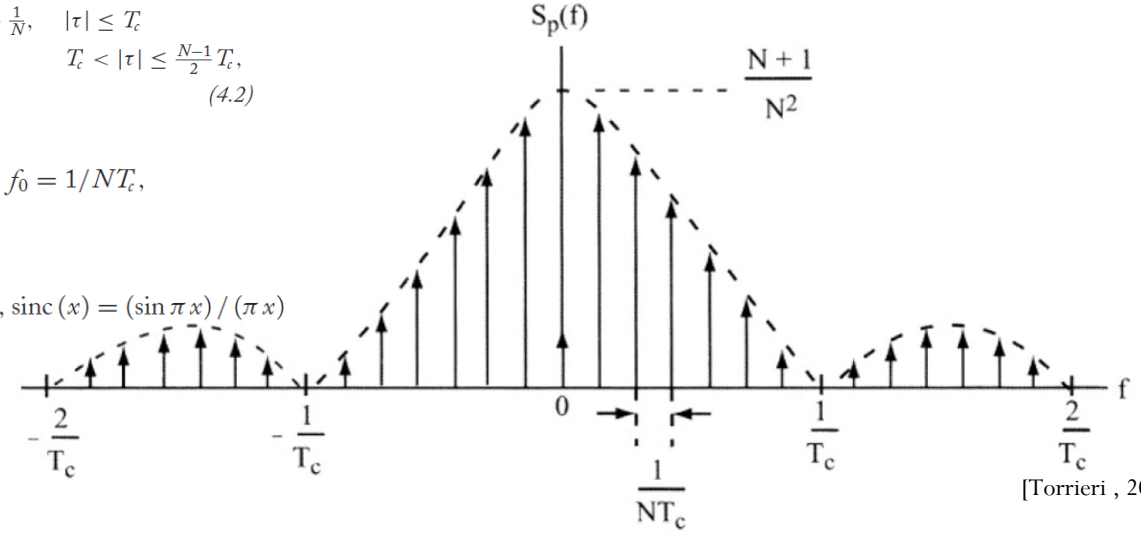
$$R_c(\tau) = \frac{1}{T_0}\int_{T_0} x(t)x(t+\tau)\,dt = \begin{cases} \left(1-\frac{|\tau|}{T_c}\right)\left(1+\frac{1}{N}\right)-\frac{1}{N}, & |\tau| \le T_c \\ -\frac{1}{N}, & T_c < |\tau| \le \frac{N-1}{2}T_c, \end{cases} \quad (4.2)$$

where the integration is over any period, $T_0 = NT_c$.

$$S_c(f) = \sum_{m=-\infty}^{\infty} P_m \delta(f - mf_0),\ f_0 = 1/NT_c,$$

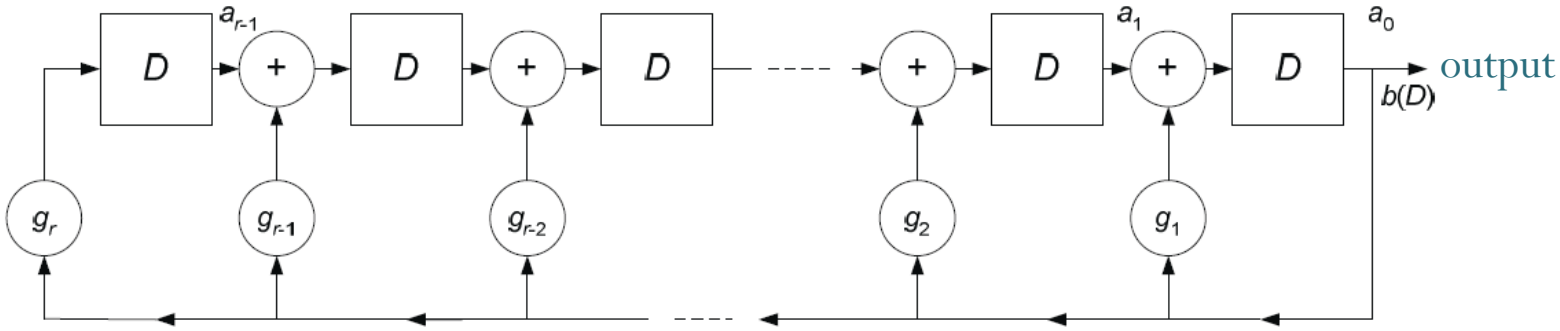
where

$$P_m = \begin{cases} \left[(N+1)/N^2\right]\operatorname{sinc}^2(m/N),\ m \neq 0,\ \operatorname{sinc}(x) = (\sin \pi x)/(\pi x) \\ 1/N^2,\ m = 0. \end{cases}$$

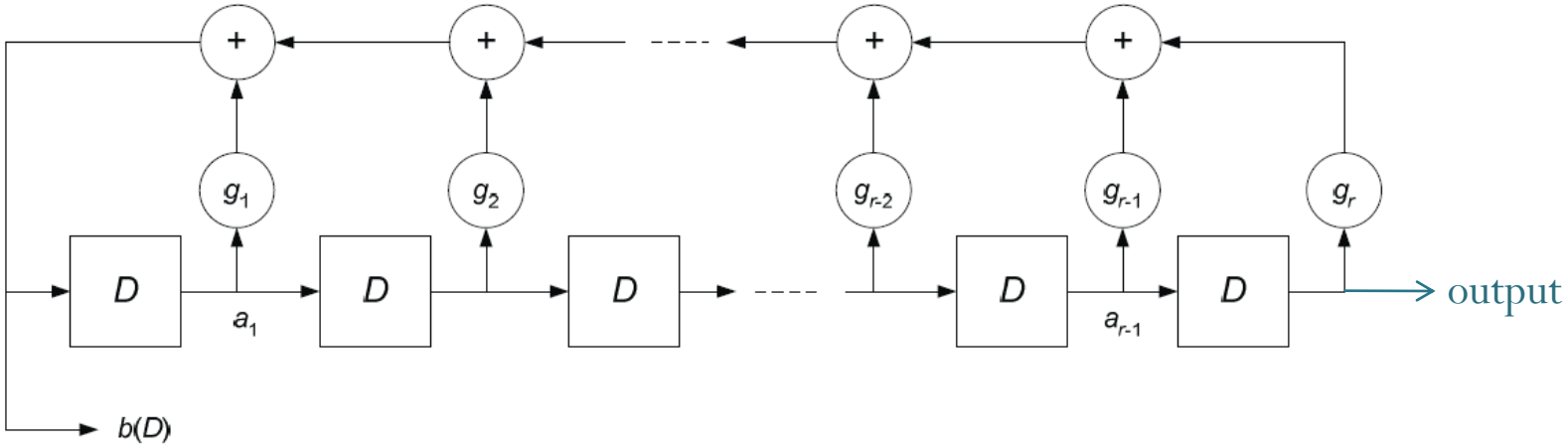


[Torrieri , 2005, Fig 2.10]

Two configurations of m-sequence generators



(a) High-speed linear feedback shift-register generator

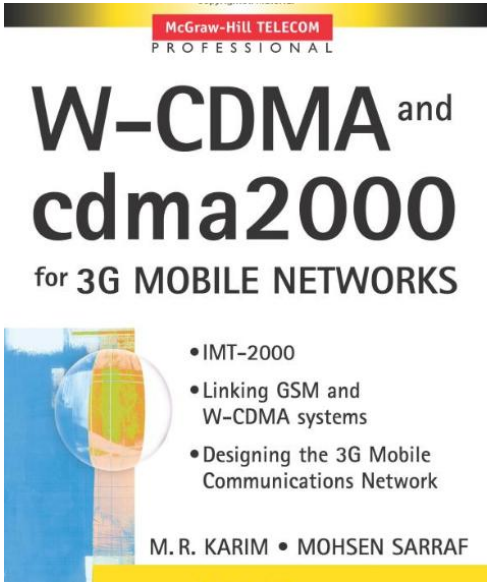


(b) Low-speed linear feedback shift-register generator (standard form)

[Ziemer, 2007, Fig. 5]
[Torrieri,2005, Fig. 2.7]

Reference

- M. R. Karim and Mohsen Sarraf, ***W-CDMA and cdma2000 for 3G Mobile Networks***, McGraw-Hill Professional, 2002.
  - Page 84-90



[TK5103.452 K37 2002]